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TRANSPORT EQUATIONS FOR A FIBROUS CONSOLIDATABLE MATERIAL AND THE NEAR-WALL LAYER EFFECT*

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The motion of a consolidatable two-phase rod, a fluid-saturated solid elastic porous cylinder, is examined in a cylindrical tube. The effect of the formation and evolution of a near-wall layer is explained qualitatively on the basis of this model. Formulas for the layer thickness and the pore pressure and obtained from the consolidation equations in one limiting case.

Unlike hydraulic transport at low concentrations, the transport of highly-concentrated fibrous materials containing 6-25% solid substance /1-3/ is realized because of the origination of a fluid near-wall layer which reduces the drag tenfold. The theory of this kind of transport has not yet been developed, and existing hydraulic transport models of low-concentration suspensions are not acceptable for this purpose. A highly-concentrated fibrous material is described below by the consolidation equations in the linear approximation.

1. The theory of linear and non-linear consolidation was developed principally in connection with questions of soil mechanics /4-8/. Without taking account of the bulk forces the linear equations of consolidation of a two-phase isotropic porous medium have the form /5/

$$G_1\Delta\mathbf{u} + G_1(1-2\nu_1)^{-1}\operatorname{grad}\operatorname{div}\mathbf{u} + (H_1 - f)\operatorname{grad}p = 0$$
$$\partial\theta/\partial t - k\Delta p = 0,\ \theta = H_2\operatorname{div}\mathbf{u} + (H_3 + H_4)p \tag{1.1}$$

Here u is the elastic displacement vector, ν_1 is Poisson's ratio, G_1 is the shear modulus, k is the filtration coefficient of the porous medium, p is the fluid pressure in its pores, t is the time, f is the porosity, i.e., the magnitude of the intercommunication pore volume per unit volume of the porous medium, the other closed pores are considered to be part of the solid phase of the skeleton (they substantially diminish the elastic moduli of both the solid phase and the medium as a whole), θ is the change in fluid content per unit volume of the medium, and H_i $(i = 1, 2, 3, 4)$ are the volume strain parameters of the porous medium, its liquid and solid phases.

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Knowing the four scalar functions satisfying the four Eqs.(1.1), the fluid filtration velocity $\mathbf{v}$ in the porous medium relative to the solid phase can be found from the Darcy formula $\mathbf{v} = -k \operatorname{grad} p$, and the elastic stress tensor is determined by a generalized Hooke's law. The physical quantities mentioned are expressed in cylindrical coordinates in the form

$$v_r = -k\frac{\partial p}{\partial r}, \quad v_\varphi = -\frac{k}{r}\frac{\partial p}{\partial \varphi}, \quad v_z = -k\frac{\partial p}{\partial z} \tag{1.2}$$
$$\sigma_r = 2G_1\left(\frac{\nu_1}{1-2\nu_1}\operatorname{div}\mathbf{u} + \frac{\partial u_r}{\partial r}\right) + H_1 p, \quad \tau_{rz} = G_1\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)$$

The stress σ_r etc., is understood to be the magnitude of the normal or tangential forces applied to unit area of the porous medium; the stresses in the skeleton are $(1-f)^{-1}$ times greater.

Several experiments are proposed /6/ to measure the parameters H_i and the following relations have been obtained: $H_1 = c + f - 1$, $H_2 = 1 - c$, $H_3 = -H_1 c_2^{-1}$, and $c = c_1 c_2^{-1}$. Here c_1 and c_2 are the coefficients of volume compression of the porous medium as an open system and solid phase material; $c_1 < c_2$. The gas contained in the fluid and in the pores of the solid substance is not considered as a separate phase. It is assumed that the fluid is incompressible compared with the gas it contains and consequently, the compressibility of the liquid gas-saturated phase is determined by the linear compressibility of the gas, i.e., $H_4 = f_0 p_0^{-1}$, where f_0 is the fraction of the gas phase in the porous medium at a total atmospheric pressure p_0, corresponding to the pore pressure $p = 0$. Since $c < 1 - f$, then $H_1 < 0$ and $H_i > 0$ for $i \geqslant 2$.

2. We examine the mechanism of rod formation and consolidation in different sections of the pipeline. New visual observations and test-stand measurements /3/ are used in selecting the model being proposed and, in particular, the boundary conditions written down below together with the results in /1-3/.

Tests showed that in connection with the high air content in highly-concentrated material (cellulose fiber in water), the pressure from the pump is transmitted initially just to its skeleton to produce large compressive stresses. The skeleton is compressed under the action of these stresses, acquires a rod shape at the entrance section of the tube and elastic properties, and part of the total pressure is perceived by the fluid. The boundary pores are closed by the tube wall and fluid does not flow out of the rod, consequently, the process of equilibration of the pressure p starts in the sections $z = \text{const}$. Therefore, the rod can be directly adjacent to the walls at the initial section of the tube. Since the walls are not deformable and the rod moves along the z axis at a velocity $a > 0$ without rotating in φ, the usual contact conditions of ultimate friction hold in this section

$$u_r = -\varepsilon, \quad \tau_{rz} = \varkappa\sigma_r, \quad \tau_{r\varphi} = 0, \quad v_r = 0 \quad (r = R) \tag{2.1}$$

Here R is the tube inner radius, $\varepsilon > 0$ is the elastic radial clearance of the rod determined by the kind of pump and the shape of the entrance end of the tube, $\varkappa > 0$ is the coefficient of friction between the rod and the tube wall, which depends on the kind and concentration of the fibre and the wall roughness; the filtration condition $v_r = 0$ means that the boundary pores are squeezed compactly to the tube and closed by its wall. If the fibre concentration or the clearance ε are too large, conditions (2.1) can turn out to be valid along the whole length of the tube; then because of the large lateral friction forces of the rod, transport will become impossible. However, as observations show, even for a 25% concentration of fibrous material a near-wall fluid layer is formed behind a short initial section between the rod and the tube wall as a result of consolidation of the porous medium. Its thickness increases initially, then remains constant, and finally in the absence of end-face constraint decreases to zero as water is absorbed by the decompressing rod. Therefore, conditions (2.1) again hold at the exist section of the pipeline for free efflux of the fibrous mass at the endface.

Let us consider the reasons for the formation of the fluid near-wall layer. For given boundary conditions at the endfaces of the rod or its individual sections the boundary value problem (1.1), (1.2), (2.1) has a unique solution but it cannot always be realized physically. Indeed, under ultimate friction conditions the fluid will not flow out of the rod $(v_r = 0)$ except in the case when the boundary pores are covered compactly by the tube walls and the pore walls are squeezed sufficiently to the tube, when the inequality $\sigma_r < -(1-f)p$ holds.

If the pore fluid turns out to be under greater pressure than the contact stresses in the skeleton $(f-1)^{-1}\sigma_r$, it penetrates between the tube walls and the pores and squeezing the rod from the tube wall, starts to increase and fill the gas that appears. Three conditions play an important part in the process of near-wall layer formation described: a) the force factor $\sigma_r = (f-1)p$; b) the connectedness of the skeleton for $r < R$, and c) the lack of connectedness (adhesion force) between the skeleton and the tube for $r = R$. Conditions b) and c) constrain the properties of materials transportable with the formation of the near-wall layer; for $\sigma_r > (f-1)p$ condition a) makes the problem (1.1), (1.2), (2.1) physically incorrect and requires that conditions (2.1) be replaced by contact boundary conditions between the rod and the near-wall layer. Observations in a transparent tube showed that the near-wall layer is a system of deep winding grooves on the rod surface, whose edges are close to the tube walls and partially abut it. The fluid flows along the grooves and over them in the laminar mode, towards lower pressures, and contains a signficant percentage of air in the bubbles whose diameters are comparable with the layer thickness and exert noticeable resistance to the motion. The fluid pressure $p = p(z, R)$ in the main section of the layer falls linearly as z increases, while the flow velocity is constant and depends only slightly on a.

Therefore, to a first approximation the flow in the layer can be considered to obey the Darcy law

$$\mathbf{w} = -k_1 \operatorname{grad} p \tag{2.2}$$

where $\mathbf{w}$ is a two-dimensional vector of the mean fluid velocity relative to a rod with the components w_z and w_φ, and k_1 is the filtration coefficient in the layer and $p = p(z, R)$. The equations of continuity and state are

$$d(\rho\Delta\Omega)/dt = 0, \quad \rho = \chi(p) \tag{2.3}$$

where ρ is the density are the aerated fluid, $\Delta\Omega$ is its volume element, $\chi(p)$ is a function of the pressure and together with (2.2) determine completely the fluid motion in the layer.

As a increases the velocity w_z falls, the layer thickness grows, and the drag in the tube drops. This is explained by the rise in the Couette flow velocity directed opposite to the flow generated by the pressure drop. In this case the Navier-Stokes equation must be used instead of (2.2).

The boundary conditions for the elastic components in the consolidation equations have the form

$$\sigma_r = -(1-f)p, \quad \tau_{rz} = \mu\partial w_z/\partial r, \quad \tau_{r\varphi} = 0 \quad (r = R) \tag{2.4}$$

where μ is the fluid coefficient of viscosity, the second condition is taken in the Beavers-Joseph form /8/, and the function $\partial w_z/\partial r$ is determined on the boundary between the rod and the near-wall layer. The missing boundary condition for the filtration components can always be obtained from (2.2) and (2.3) by writing them for velocities, pressures, and densities averaged over the layer thickness.

3. We examine one elementary solution of problem (1.1), (1.2) ,(2.2), (2.4) by assuming that consolidation of the fibrous material terminates in the section $0 \leqslant r \leqslant R,\ 0 \leqslant z \leqslant 1,\ p = \text{const}$, the outflowing fluid remains in the near-wall layer of this section, there is no friction in the layer and no initial rod tension $\gamma = \varepsilon = 0$. Such conditions are satisfied in a compression device with impermeable walls and, approximately, in the middle part of the pipeline section. Let the magnitude of the rod compression $\delta > 0$ be given

$$u_z = -\delta, \quad \tau_{rz} = \tau_{\varphi z} = 0 \quad (z = 1), \quad u_z = \tau_{rz} = \tau_{\varphi z} = 0 \quad (z = 0), \quad 0 \leqslant r \leqslant R \tag{3.1}$$

Find the thickness of the fluid near-wall layer $h = -u_r$ for $r = R$. If $p = \text{const}$, then the first equation in (1.1) goes over into the Lamé equation, the second equation in (1.1) and codition (2.2) are satisfied identically. To satisfy the boundary conditions (2.4) and (3.1) we take the following axisymmetric solution of the elasticity theory problem: $u_\varphi = 0,\ u_r = (A + \delta\nu_1)r,\ u_z = -\delta z$. Determining the constant A and p from the first condition in (2.4) and the condition for the displaced fluid volume $-\pi R^2\theta$ being equal to the near-wall layer volume $-2\pi R u_r$, we obtain by virtue of (1.1) and (1.2)

$$h = R\delta\,[(1-c)(1-2\nu_1) + 3\nu_1(f - c_2H_4)]/N \tag{3.2}$$

$$p = 3\delta c_2\,[(1-c)(1-2\nu_1) + 2\nu_1]/N, \quad N = 3(c_2H_4 - H_1) + 2(1+\nu_1)c \tag{3.3}$$

In the denominators and in the numerator of (3.3) each component is positive, only the first component in the numerator of (3.2) is positive. Therefore, $p > 0$, and the formulation of the near-wall layer $(h > 0)$ depends on relationships between ν_1, f, c_1, c_2, H_4. If $\nu_1 = 0$, then the layer is formed for any other parameters of the problem. Its thickness is proportional to R and δ, and increases as ν_1, H_4, c_2 decrease and as f increases.

4. The results of an analysis of the transport model considered agree qualitatively with test data. Unlike the hydraulic transport of low-concentration material with a parabolic velocity distribution diagram corresponding to Poiseuille flow, the fluid had a higher velocity in the near-wall layer than the rod $w_z > 0$ over the whole range of rod velocities a in our tests. This also follows from condition (2.2). The drop in pipeline drag as R increases /2/ can be explained by the fact that according to (3.2) the quantity h is proportional to R. The observable stable increase in the layer thickness h as the pressure p and the longitudinal strains δ increase is in agreement with (3.2) and (3.3). Finally, the drop in drag when air is injected into the pipeline /2/ follows from (3.2) as h increases for a decrease in c_2 and an increase in f.

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ON A DYNAMIC CONTACT PROBLEM FOR A SINGLE ELECTRODE*

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The dynamic problem of surface-wave excitation by the main element of electrode transducers, a single electrode simulated by a strip stamp lying freely on the surface of a piezoelectric half-space, is considered. The vertical component of the displacement and the electrical potential is given in the contact region, while the surface outside this region has no electrical and mechanical loads. The boundary value problem of electroelasticity mentioned reduces to investigating a system of inhomogeneous Fredholm type integral equations of the first kind in the unknown normal stress and charge distribution density functions.

The regularization method for the system of integral equations obtained is based on constructing the factorization of the kernel matrix-function and enables the system of integral equations of the first kind to be reduced to a system of integral equations of the second kind with a completely continuous operator for which separation into finite-dimensional and small terms is effective. Solutions are obtained for this system, that describe the behaviour of the contact stresses and the charge distribution density on the electrode, as well as the displacement and potential wave fields on the free piezoelectric surface, with the assignment of the electrical and mechanical perturbations taken into account. The absolute values of the deviation of the excited wave phase velocity from the Rayleigh wave velocity are computed at given points on the ST-cut surface of a piezoelectric quartz crystal.

Approaches developed earlier for constructing approximate solutions of the problems of the excitation and interaction of surface waves with metallic electrodes are based, as a rule, on the assumption of the weightlessness of the electrodes without taking account of the influence of the mechanical perturbations and the nature of the contact with the medium /1-3/. At high frequencies as well as during examination of resonators these factors are of no little importance.

1. We introduce an $Ox_1x_2x_3$ coordinate system and we assume that the crystal occupies the domain $x_3 \leqslant 0$; x_1 is the wave propagation direction and the electrode dimensions along the x_2 axis are infinite.

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